Q.1 For a real number \( x \), let \( [x] \) denote the greatest integer less than or equal to \( x \). Let \( f: \mathbb{R} \to \mathbb{R} \) be the function defined by

\[
  f(x) = \begin{cases} 
    \frac{([x^2] + [x^2 - 1]) \sin^3 x}{x^3} & \text{if } x \neq 0, \\
    k & \text{if } x = 0.
  \end{cases}
\]

Then the value of \( k \) for which \( f \) is continuous at 0 is

(A) 2    (B) −1    (C) 0    (D) 1

Q.2 Let \( z_1 \) and \( z_2 \) be complex numbers such that \( i(z_1 - z_2) \) and \( z_1 + z_2 \) are both real numbers, where \( i^2 = -1 \). Which one of the following statements is true?

(A) \( z_1 = \overline{z_2} \)    (B) \( z_1 = z_2 \)    (C) \( z_1 = i \overline{z_2} \)    (D) \( z_1 = -\overline{z_2} \)

Q.3 A heap of sand is in the form of a cone whose height is 9 meters and it contains \( 432 \pi \) cubic meters of sand. The minimum area, in square meters, of the canvas required to cover the heap is

(A) \( 12 \sqrt{63} \pi \)    (B) 324 \pi    (C) 180 \pi    (D) 192 \pi

Q.4 The number of one-one (injective) functions from the set \( A = \{-1, 0, 1\} \) to the set \( B = \{1, 2, 3, 4\} \) is

(A) 12    (B) 81    (C) 64    (D) 24

Q.5 The value of \( \theta \) in the interval \( \left[0, \frac{\pi}{2}\right] \) satisfying \( 1 + \cos \theta + \cos^2 \theta + \cos^3 \theta + \cdots = 4 + 2 \sqrt{3} \) is

(A) \( \frac{\pi}{2} \)    (B) \( \frac{\pi}{3} \)    (C) \( \frac{\pi}{4} \)    (D) \( \frac{\pi}{6} \)

Q.6 The number of possible \( 3 \times 3 \) matrices with entries from the set \( \{0, 1\} \) is

(A) 8    (B) 18    (C) 512    (D) 81
Q.7 Let \( f : \mathbb{R} \to \mathbb{R} \) be a two times differentiable function such that \( f''(x) \geq 0 \) for every \( x \in \mathbb{R} \) and \( f'(0) > 0 \), where \( f' \) and \( f'' \) denote the first and the second order derivatives of \( f \), respectively. If \( g(x) = f(e^x) \), then

(A) \( g''(x) > 0 \) for all \( x \in \mathbb{R} \).

(B) \( g''(x) = 0 \) for some \( x \in \mathbb{R} \).

(C) \( g''(x) < 0 \) for all \( x \in \mathbb{R} \).

(D) There exist \( x_1 \) and \( x_2 \) in \( \mathbb{R} \) such that \( g''(x_1) < 0 \) and \( g''(x_2) > 0 \).

Q.8 For which one of the following values of \( p \), does the following inequality hold for all \( x \geq 0 \)?

\[
(1 + x)^p \leq 1 + x^p
\]

(A) \( p = \frac{3}{2} \) \hspace{1cm} (B) \( p = \frac{1}{2} \) \hspace{1cm} (C) \( p = 2 \) \hspace{1cm} (D) \( p = 4 \)

Q.9 For which one of the following values of \( k \), the equation

\[
\cos \left(\frac{k\pi}{7}\right) = \cos \left(\frac{6\pi}{7}\right)
\]

is satisfied?

(A) 9 \hspace{1cm} (B) 5 \hspace{1cm} (C) 8 \hspace{1cm} (D) 4

Q.10 The value of the sum

\[
\sum_{n=1}^{2024} \frac{1}{\sqrt{n} + 1 + \sqrt{n}}
\]

is

(A) 24 \hspace{1cm} (B) 45 \hspace{1cm} (C) 20 \hspace{1cm} (D) 44
SECTION 2 SDI

- This section contains TEN (10) questions.
- The answer to each question is a SINGLE DIGIT NON-NEGATIVE INTEGER (SDI).
- Answer to each question will be evaluated according to the following marking scheme:
  
  \( \text{Full Marks} : +4 \) if ONLY the bubble corresponding to the correct answer is darkened.
  
  \( \text{Zero Marks} : 0 \) In all other cases.

Q.11 The number of real solutions of the equation

\[
\sqrt{x^2 - 1} \left( x^2 - 2x + \frac{3}{4} \right) = 0
\]

is ______

Q.12 A boy of height 1.2 meters walks at a rate of 28 meters per minute away from a lamp which is 4 meters above the ground. If \( k \) meters per minute is the rate at which the length of the shadow of the boy is increasing, then the value of \( \frac{k}{4} \) is ______

Q.13 The value of \( m \) for which the points \( A = (-3, 1, -1) \), \( B = (1, m, 1) \), and \( C = (-1, 2, 0) \) are collinear is ______

Q.14 For a complex number \( z \), let \( \text{Re}(z) \) denote its real part. If \( z_1 \) and \( z_2 \) are the non-real roots of the equation \( (z - 1)^3 - 1 = 0 \), then the value of \( \text{Re}(z_1) + \text{Re}(z_2) \) is ______

Q.15 The number of values of \( \theta \) in the interval \( [0, \frac{\pi}{2}] \) satisfying the equation

\[
\sin 2\theta - \cos 2\theta = 1 + \sin \theta - \cos \theta
\]

is ______

Q.16 While entering data consisting of 10 numbers, a person makes a mistake of entering a two-digit number in the reverse order. In doing so the mean of the data decreases by 1.8. Then the absolute value of the difference of the digits of the two-digit number is ______

Q.17 Out of a group of 6 girls and 4 boys, a team of three is formed at random. Let \( p \) be the probability that the team consists of 2 girls and 1 boy. Then the value of \( 4p \) is ______

Q.18 Let

\[
f(x) = \int_{-1}^{x} t|t|dt.
\]

Then the value of \( 3f(2) \) is ______
Q.19 Let $S$ be the area of the region in the first quadrant enclosed by the three lines $x = 0, y = 0, x = 2$, and the graph of the function

$$f(x) = \begin{cases} 
2 & \text{if } 0 \leq x \leq 1, \\
\frac{2}{x^2} & \text{if } x > 1.
\end{cases}$$

Then the value of $S$ is ______

Q.20 Consider the $3 \times 3$ matrices $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Then the trace of the matrix

$$(A^3 - 6I)$$

is ______
SECTION 3: Paragraph based MCQ

This section contains FIVE (05) paragraphs.

Based on each paragraph, there are TWO (02) questions.

Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.

If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.

Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the bubble corresponding to the correct answer is darkened.
Zero Marks : 0 If none of the bubble is darkened (i.e., the question is unanswered).
Negative Marks : −0.75 In all other cases.

PARAGRAPH I

Consider the triangle $\triangle ABC$ with sides $AB$, $BC$, $CA$ in the ratio $1 : r : r^2$ for some $r > 0$. Suppose that the angle $\angle BCA = \alpha$ and the angle $\angle ABC = 2\alpha$.

Q.21 Which one of the following statements is true?

(A) $0 < \alpha < \frac{\pi}{6}$ (B) $\frac{\pi}{5} < \alpha < \frac{\pi}{3}$ (C) $\frac{\pi}{6} < \alpha < \frac{\pi}{5}$ (D) $\frac{\pi}{3} < \alpha < \frac{\pi}{2}$

PARAGRAPH I

Consider the triangle $\triangle ABC$ with sides $AB$, $BC$, $CA$ in the ratio $1 : r : r^2$ for some $r > 0$. Suppose that the angle $\angle BCA = \alpha$ and the angle $\angle ABC = 2\alpha$.

Q.22 Which one of the following statements is true?

(A) $r^2 = \cos \alpha$ (B) $r^2 = \sin \alpha$
(C) $r^2 = 2\cos \alpha$ (D) $r^2 = \cos 2\alpha$

PARAGRAPH II

A fair coin is tossed three times. Let $E_1$ be the event that exactly 2 heads appear, $E_2$ be the event that exactly 1 head appears, and $E_3$ be the event that at least 1 head appears.

Q.23 The conditional probability $P(E_2|E_3)$ is

(A) $\frac{3}{7}$ (B) $\frac{3}{8}$ (C) $\frac{4}{7}$ (D) $\frac{5}{8}$
PARAGRAPH II

A fair coin is tossed three times. Let $E_1$ be the event that exactly 2 heads appear, $E_2$ be the event that exactly 1 head appears, and $E_3$ be the event that at least 1 head appears.

Q.24 The conditional probability $P(E_1|E_3)$ is

(A) $\frac{3}{8}$  
(B) $\frac{3}{7}$  
(C) $\frac{4}{7}$  
(D) $\frac{5}{8}$

PARAGRAPH III

Let $L$ be the normal to the parabola $y^2 = 16x$ at the point $(1, 4)$ on the parabola.

Q.25 The $y$-intercept of the line $L$ is

(A) $\frac{3}{2}$  
(B) 9  
(C) 7  
(D) $\frac{9}{2}$

PARAGRAPH III

Let $L$ be the normal to the parabola $y^2 = 16x$ at the point $(1, 4)$ on the parabola.

Q.26 The $x$-intercept of the line $L$ is

(A) $\frac{3}{2}$  
(B) 9  
(C) 7  
(D) $\frac{7}{2}$

PARAGRAPH IV

Let $\vec{u}$ and $\vec{v}$ be vectors such that $|\vec{u}| = 1$, $|\vec{v}| = 2|\vec{u}|$, and $\vec{u}$ is perpendicular to $\vec{u} + \vec{v}$. Let $\vec{w}$ be a vector perpendicular to both $\vec{u}$ and $\vec{v}$ such that $\vec{w} \cdot (\vec{u} \times \vec{v}) = 6$.

Q.27 Which one of the following statements is true?

(A) $|\vec{w}| = 2\sqrt{3}$

(B) $\vec{w}$ is perpendicular to $\vec{u} \times \vec{v}$

(C) $\vec{w}$ is parallel to $\vec{u} + \vec{v}$

(D) $|\vec{w}| = \frac{\sqrt{3}}{2}$
Then the angle between \( \mathbf{u} \) and \( \mathbf{v} \) is

(A) \( \frac{\pi}{3} \) \hspace{1cm} (B) \( \frac{2\pi}{3} \) \hspace{1cm} (C) \( \frac{3\pi}{4} \) \hspace{1cm} (D) \( \frac{5\pi}{6} \)

Which one of the following statements is true?

(A) \( g \) is one-one, but \( f \circ g \) is not one-one.

(B) There exists \( x \neq 0 \) such that \( f(g(x)) = 1 \).

(C) For any real number \( \alpha \geq 2 \), there exists \( x \in \mathbb{R} \) satisfying \( f(g(x)) = \alpha \).

(D) \( \text{Range}(f \circ g) = \text{Range}(g) \).

The domain of the function \( f \) is

(A) \([-2, 2]\) \hspace{1cm} (B) \((\infty, -2] \cup [2, \infty)\)

(C) \([2, \infty)\) \hspace{1cm} (D) \((\infty, -2]\)